

experimentally, in order to completely describe the pressure-time history in the compression chamber, and hence, the pressure-release time T_R .

In an effort to express the time constant θ_1 analytically, the following analysis is made. A general expression for the equation of motion of a viscous, compressible fluid is derived in reference (c) and can be expressed in vector form as

$$\rho \frac{d\bar{v}}{dt} = \rho \bar{F} - \nabla P - \frac{2}{3} \nabla [\mu (\nabla \cdot \bar{v})] + [\nabla \cdot (2\mu \nabla)] \bar{v} + \nabla \times (\mu \bar{\zeta}) \quad (7)$$

where the nomenclature is

- ρ . . . mass density of the fluid, slugs/in³
- \bar{v} . . . velocity vector, in/sec
- P . . . fluid pressure, psia
- \bar{F} . . . body forces per unit mass, lb/slug
- μ . . . coefficient of viscosity of fluid, lb-sec/in²
- $\bar{\zeta}$. . . vorticity vector, sec⁻¹
- ∇ . . . operator del

The vorticity vector $\bar{\zeta}$ is defined as the curl of the velocity vector

$$\bar{\zeta} = \nabla \times \bar{v} \quad (8)$$

Combining (8) with (7) and performing the operations indicated by the operator ∇ , we have

$$\begin{aligned} \rho \frac{d\bar{v}}{dt} = & \rho \bar{F} - \nabla P - \mu \nabla \times \bar{\zeta} + \frac{4}{3} \mu \nabla (\nabla \cdot \bar{v}) \\ & + 2(\nabla \cdot \nabla \mu) \bar{v} + \nabla \mu \times \bar{\zeta} - \frac{2}{3} \mu (\nabla \cdot \bar{v}) \end{aligned} \quad (9)$$

If we invoke the condition of incompressibility

$$\nabla \cdot \bar{v} = 0 \quad (10)$$

and if we consider the coefficient of viscosity to be independent of the spatial coordinates

$$\nabla \mu = 0 \quad (11)$$

then equation (9) is simplified to the form

$$\rho \frac{d\bar{v}}{dt} = \rho \bar{F} - \nabla P - \mu \nabla^2 \bar{v} \quad (12)$$

Combining this with equation (8), we have

$$\frac{d\bar{v}}{dt} = \bar{F} - \frac{1}{\rho} \nabla P + \nu \nabla^2 \bar{v} \quad (13)$$

where

ν . . . is kinematic viscosity of fluid = $\frac{\mu}{\rho}$, in²/sec

Expanding equation (13) into its components in cylindrical coordinates, we obtain the familiar Navier-Stokes equations for a viscous, incompressible fluid

$$\begin{aligned} \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial r} + \frac{v}{r} \frac{\partial u}{\partial \theta} + w \frac{\partial u}{\partial z} - \frac{v^2}{r^2} &= F_1 - \frac{1}{\rho} \frac{\partial P}{\partial r} \\ + \nu \left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} + \frac{1}{r^2} \frac{\partial^2 u}{\partial \theta^2} + \frac{\partial^2 u}{\partial z^2} - \frac{u}{r^2} - \frac{2}{r^2} \frac{\partial v}{\partial \theta} \right) & \end{aligned} \quad (14a)$$